Median and Mean

Concepts

1. The **mean** of a continuous random variable is the same as the expected value and is given by

$$\mu = E[X] = \int_{-\infty}^{\infty} x f(x) dx.$$

A CDF is a function $F(x)$ where $F(x) = P(X \leq x)$, it tells us that probability of getting a value less than or equal to x. It is just defined as $F(x) = \int_{-\infty}^{x} f(x) dx$. It satisfies three important properties:

- $F(x)$ is nondecreasing. So if $x \leq y$, then $F(x) \leq F(y)$.
- $\lim_{x \to -\infty} F(x) = 0$.
- $\lim_{x \to \infty} F(x) = 1$.

The **median** is the point that is at the midpoint of the probability distribution. It is when $P(X \leq x) = 0.5$ or when the CDF is equal to 0.5.

Example

2. Let $g(x) = \begin{cases} x & 0 \leq x \leq 1 \\ 2 - x & 1 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$. Find c such that $f(x) = cg(x)$ is a PDF. Graph f and the CDF F. Find the mean and median of $f(x)$.

Problems

3. True   False   It is possible for the mean for a discrete PDF to not exist.

4. True   False   Another name for the mean of a PDF is the expected value.

5. True   False   For a discrete PDF, the mean occurs with nonzero probability.

6. True   False   There exists a uniform distribution on all the real numbers.

7. Let $g(x) = \begin{cases} x^2 & -1 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$. Find c such that $f(x) = cg(x)$ is a PDF. Graph f and the CDF F. Find the mean and median of $f(x)$.

8. Let $F(x) = \frac{x-1}{x+1}$ for $x \ge 1$ and 0 for $x \le 1$. Show that F is a CDF. Find the PDF associated with it and the probability that we choose a number between 1 and 2.

9. Let $g(x) = \begin{cases} e^{-x} & -1 \le x \\ 0 & \text{otherwise} \end{cases}$. Find c such that $f(x) = cg(x)$ is a PDF. Graph f and the CDF F. Find the mean and median of $f(x)$.

10. Let $g(x) = \begin{cases} \frac{1}{x^4} & x \le -1 \\ 0 & \text{otherwise} \end{cases}$. Find c such that $f(x) = cg(x)$ is a PDF. Graph f and the CDF F. Find the mean and median of $f(x)$.

11. Let $g(x) = \begin{cases} \frac{1}{x^4} & 2 \le x \\ 0 & \text{otherwise} \end{cases}$. Find c such that $f(x) = cg(x)$ is a PDF. Graph f and the CDF F. Find the mean and median of $f(x)$.